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Hydraulic diffusion in unsaturated material

Example of User module to model unsaturated flow

The flow in the matrix is governed by the Darcy's law:

$$\underline{v} = - \frac{\mathbf{k}^{in}}{\mu} k_r(S) \nabla(p + \gamma_w z) \quad (1)$$

with \underline{v} the fluid velocity, p the pressure, μ the dynamic viscosity of the fluid, \mathbf{k}^{in} the intrinsic permeability tensor and k_r the relative permeability function of the *saturation degree* S :

$$k_r(S) = \sqrt{S} \left(1 - (1 - S^{1/m'})^{m'} \right)^2 \quad (2)$$

m' is a material constant (See also [1]). γ_w is the water pressure gradient given in the general parameters of the model (Problem Type).

The mass balance equation in the matrix reads:

$$\text{div}(\rho_f \underline{v}) + \partial m_f / \partial t = 0 \quad (3)$$

where m_f is the fluid mass present in the unit volume of the matrix and ρ_f the fluid specific mass. In unsaturated conditions:

$$m_f = \rho_f \phi S \quad (4)$$

with ϕ the porosity and S saturation degree.

The variation of ϕ is given by the poromechanical model of the matrix and is related. But here we consider a pure hydraulic model and therefore *assume the porosity constant*, so $d\phi=0$. For an extension to poroelastic behavior the equation $d\phi = a_m d\varepsilon_v + a_h dp$ from the paper [2] can be used.

To make possible a standard formulation of the the governing equations for numerical modelling by FEM, the variation of m_f is to be related to that of the nodal variable p . Starting with:

$$\frac{dm_f}{m_f} = \frac{d\rho_f}{\rho_f} + \frac{dS}{S} \quad (5)$$

The variation of ρ_f is related to that of p by:

$$\frac{d\rho_f}{\rho_f} = \frac{dp}{K_f} \quad (6)$$

where K_f is the fluid compressibility.

For the saturated material, $S=1$ and the pressure p takes positive values. For unsaturated material, $S<1$ and its variation is function of the *suction* s as given by the *retention curve*. The suction s is the difference between the gas (air, vapor) pressure p_g and the fluid pressure p in the pore space: $s = p_g - p$. In UNSAT material, only two phases, solid skeleton and pore fluid are modeled, and then, if a gas phase exists, it is considered to be at constant zero pressure. This means that, in unsaturated state, $s = -p$. By extending the pressure values to the negative domain it is possible to represent with a unique variable p the pressure in saturated conditions ($p \geq 0$) and the suction in unsaturated conditions ($p < 0$). The saturation degree is related to this variable by a function $S = f(s) = f(-p)$ coinciding with the retention curve in unsaturated conditions ($p < 0$) and extended by $S=1$ for $p \geq 0$ (see the Figure 1).

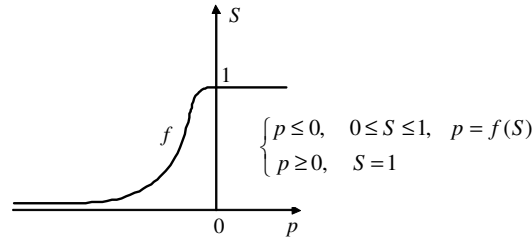


Figure 1. The retention curve f of the porous material extended to the saturated phase with $p > 0$, $S=1$

The time derivatives of S and p are then related by:

$$\frac{\partial S}{\partial t} = S' \frac{\partial p}{\partial t} \quad (7)$$

where:

$$\begin{cases} S' = 0 & \text{if } p \geq 0, \quad S = 1 \\ S' = -\frac{df}{ds} & \text{if } p \leq 0, \quad 0 \leq S \leq 1 \end{cases} \quad (8)$$

If, for instance, the Van Genuchten [3] law is chosen for the retention curve:

$$S = \frac{1}{(1 + (\alpha s)^n)^m} \quad (9)$$

with $\alpha > 0$, $0 \leq m < 1$ and $n = (1-m)^{-1}$. Then, for $p < 0$:

$$S' = \frac{-mnS}{p} (1 - S^{1/m}) \quad (10)$$

Replacing by the equations (1),(5),(6) and (7) in (3), the following governing equation is obtained for the pressure field evolution:

$$\text{div}(\mathbf{K} \nabla p) = C \frac{\partial p}{\partial t} \quad (11)$$

with:

$$K = \frac{\rho_f}{\mu} k^{in} k^r, \quad C = \rho_f \phi \left(\frac{S}{K_f} + S' \right) \quad (12)$$

The assumption of incompressible fluid ($\rho_f = cste.$) brings further simplification. We can simply ρ_f from the two sides and write :

$$K = \frac{k^{in}}{\mu} k^r, \quad C = \phi \left(\frac{S}{K_f} + S' \right) \quad (13)$$

The parameters to be specified for the materials are thus $k = k^{in}/\mu$, ϕ , K_f and the parameters m, n, α for the Van Genuchten law as well as the parameter m' for the relative permeability law. The water pressure gradient γ_w is given in the general parameters (Problem Type).

Nb = 7

Param1 = $k = k^{in}/\mu$ (permeability)

Param2 = ϕ (porosity)

Param3 = K_f (water compressibility)

Param4 = m' (for relative permeability)

Param5 = m (for Van Genuchten)

Param6 = n (for Van Genuchten)

Param7 = α (for Van Genuchten)

Internal variable:

$V_h^{in}(n,1)$: internal variable 1-S

Appendix: Some examples of parameters values

From : Giulia Bella (2021). 'Water retention behavior of tailings in unsaturated conditions', Geomechanics and Engineering 26(2):15

Sample	Main drying branch			Main drying branch			AEV (kPa)
	α (kPa ⁻¹)	n (-)	m (-)	α (kPa ⁻¹)	n (-)	m (-)	
SLURRY-0.78	0.030	1.630	0.390	0.280	1.480	0.320	33.3
SILT-0.70	0.042	1.620	0.380	0.320	1.500	0.330	23.8
SILT-0.60	0.019	1.650	0.390	0.100	1.520	0.340	52.6
SILT-0.50	0.009	1.670	0.400	0.065	1.520	0.340	111.1
7030-0.70	0.120	1.550	0.350	0.600	1.700	0.410	8.3
3070-0.60	0.021	1.680	0.400	0.150	1.540	0.350	47.2

References

[1] Disroc Materials Catalogue, model Gelisol 32111

(<http://www.fracsim.com/DISROC/Materials-Catalog.pdf>)

[2] Pouya A., A finite element method for modeling coupled flow and deformation in porous fractured media. Int. J. Numer. Anal. Meth. Geomech. 2015; 39:1836–1852, DOI: 10.1002/nag.2384

[3] Van Genuchten, M. T. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal, 1980; **44** (5): 892-898.